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A METHOD OF CALIBRATING AIRSPEED INSTALLATIONS ON AIRPLANES AT TRANSONIC AND SUPERSONIC SPEEDS BY USE OF ACCELEROMETER AND ATTITUDE-ANGLE MEASUREMENTS

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SUMMARY

A method is described for calibrating airspeed installations on airplanes at transonic and supersonic speeds in vertical-plane maneuvers in which use is made of measurements of normal and longitudinal accelerations and attitude angle. The method involves starting or ending a calibration run near level flight at a speed for which the airspeed calibration is known and hence for which the free-stream static pressure may be determined. Integration of vertical velocity, computed from the normal and longitudinal accelerations and the attitude angle, determines the change in altitude which, when combined with the temperature measurements, gives the change in free-stream static pressure from the start or end of the calibration run and hence the variation of free-stream static pressure during the calibration run. In this method, all the required instrumentation is carried within the airplane.

A study of the effect of the various sources of error on the accuracy of a calibration indicated that the quantities can be measured accurately enough to insure a satisfactory calibration.

INTRODUCTION

A method of calibrating the static-pressure source of a pitot-static installation on an airplane at high speed and high altitude by the use of radar-phototheodolite tracking equipment was described in reference 1. In this method, the radar-phototheodolite equipment is used to establish the geometric altitude of the airplane in surveys of atmospheric pressure made at speeds for which the airspeed calibration is known and in maneuvers under conditions for which the calibration is desired. Although this method is precise, its use is limited since radar-phototheodolite equipment is not generally available. In order that airspeed calibrations

may be performed universally, two additional methods were devised at the Langley Aeronautical Laboratory. One method, described in detail in reference 2, makes use of measurements of total pressure, static pressure, and temperature. In this method, a survey is made of static pressure and temperature at airplane speeds at which the airspeed calibration is known. The airplane is then flown through the region surveyed under conditions for which the calibration is desired and measurements of total pressure, static pressure, and temperature are repeated. The values of total pressure and temperature at a given instant in the calibration run, together with the calibration constant of the thermometer, are used to determine the relation between ambient temperature and static pressure. This relation, when compared with that for the survey, determines the free-stream static pressure at that instant. This method requires precise measurements of temperature. Another method, described herein, makes use of measurements of acceleration and attitude angle together with measurements required for the previous method. The accuracy of this method is determined principally by the accuracy of the measurements of normal and longitudinal accelerations and of the attitude angle. Measurements of temperature need not be so precise as those for the previous method. The method is restricted to vertical-plane maneuvers.

SYMBOLS

p	free-stream static or atmospheric pressure
p_m	indicated static pressure
ρ	density
e_p	static-pressure error $\left(\frac{p_m - p}{p}\right)$
Δp_m	change of static pressure indicated by statoscope recorder
p_T	free-stream total pressure for subsonic flow and total pressure behind normal shock for supersonic flow
t	elapsed time
t_m	measured elapsed time
e_t	error in elapsed time $\left(\frac{t_m - t}{t}\right)$

h	altitude
M	free-stream Mach number
M'	indicated free-stream Mach number
T	free-stream temperature, absolute units
T_m	measured temperature, absolute units
T'	temperature defined by equation (5)
e_T	error in free-stream temperature $\left(\frac{T' - T}{T}\right)$
K	temperature recovery factor $\left(\frac{T_m - T}{0.2M^2T}\right)$
a_x	longitudinal acceleration, positive forward along x-axis
a_z	normal acceleration, positive upward along z-axis
a_v	vertical acceleration, positive upward along vertical of earth (defined by equation (10))
a_y	lateral acceleration, positive to right of y-axis
α	angle between rays of sun and longitudinal axis as measured by sun camera
β	longitude of airplane
γ	ratio of specific heats (1.4)
ϵ	declination of sun
λ	elevation angle of sun
θ	attitude angle, positive below horizon
τ	latitude of airplane
ϕ	angle of bank, positive right wing down
ψ	angle of yaw, positive to right

ω Greenwich hour angle

v vertical velocity

R gas constant

s_β error in estimating latitude of airplane (distance along longitude β)

s_τ error in estimating longitude of airplane (distance along latitude τ)

g acceleration due to gravity

Δ prefix denoting error

Subscript:

1 near start or end of maneuver

METHOD

The principal feature of most procedures for calibration of airspeed installations on airplanes is the determination of ambient or free-stream static pressure. In the method described herein, free-stream static pressure is obtained over a range of altitude with the aid of the relation that the rate of change of static pressure with altitude is equal to the density, or

$$\frac{dp}{dh} = -g\rho \tag{1}$$

Since

$$g\rho = \frac{p}{RT}$$

then

$$\frac{dp}{dh} = -\frac{p}{RT}$$

or

$$dp = -\frac{p}{RT} dh \qquad (2)$$

Equation (2) may be integrated as

$$p = p_1 e^{-\int_{h_1}^{h} \frac{dh}{RT}} \qquad (3)$$

Therefore, if the pressure at one altitude is known, the pressure at other altitudes may be determined provided the ambient temperature and the change in altitude are determined. The ambient temperature T may be determined from the measured temperature T_m with the use of the relation

$$T = \frac{T_m}{1 + \frac{\gamma - 1}{2} KM^2} \qquad (4)$$

Since only the indicated Mach number M' is known for flight conditions where no airspeed calibration exists, the ambient temperature is determined only approximately as

$$T' = \frac{T_m}{1 + \frac{\gamma - 1}{2} KM'^2} \qquad (5)$$

The use of T' in equation (3) would result in a small error in p and hence two or more approximations may be necessary.

An alternate integral form of equation (2) is

$$\left(\frac{p}{p_1}\right)^n = 1 - n \int_{h_1}^{h} \left(\frac{p}{p_1}\right)^n \frac{dh}{RT} \qquad (6)$$

After substitution of the measured pressure p_m and the temperature T' in the right side of equation (6), the equation becomes

$$\left(\frac{p}{p_1}\right)^n = 1 - n \int_{h_1}^{h} \left(\frac{p_m}{p_1}\right)^n \frac{dh}{RT'}$$

or

$$\left(\frac{p}{p_1}\right)^n = 1 - n \int_{h_1}^{h} \left(\frac{p_m}{p_1}\right)^n \frac{\left(1 + \frac{\gamma - 1}{2} KM'^2\right)}{RT_m} dh \tag{7}$$

The value of n may be selected for the flight conditions encountered so that only one approximation is required in the determination of p. (See appendix A.) For values of temperature recovery factor K of the thermometer near unity and for subsonic and low-supersonic airspeeds, a value of n of $\frac{\gamma - 1}{\gamma}$ or 0.286 gives satisfactory results.

The change in altitude dh in equation (7) may be determined from vertical velocity computed from measurements of pressure and temperature at an instant in the calibration run when the airspeed calibration or the static pressure is known and from vertical acceleration computed from measurements of longitudinal and normal acceleration and attitude angle, as

$$dh = \left(v_1 + \int_{t_1}^{t} a_v \, dt\right) dt \tag{8}$$

where

$$v_1 = \frac{-RT_1}{p_1} \left(\frac{dp}{dt}\right)_1 \tag{9}$$

and

$$a_v = a_z \cos \theta - a_x \sin \theta - g \tag{10}$$

The method as outlined here is limited to maneuvers in vertical planes. The method is further illustrated by a hypothetical calibration in appendix D.

The relation given by equation (7) may also be used to advantage in the radar method of reference 1, particularly for tests which would not permit a survey of static pressure to be made over the desired range of altitude at flight conditions (Mach number and lift coefficient) at which the airspeed calibration was known. For such a flight condition, however, the airspeed calibration at least at one instant during the test must be known and the change in altitude from the altitude at that instant would, of course, be determined from radar measurements.

EQUIPMENT

The airplane on which the pitot-static installation is to be calibrated should be equipped with a recording altimeter, an airspeed recorder, a statoscope with a sensitive pressure recorder, a thermometer and temperature recorder, an accelerometer with normal and longitudinal components, an attitude recorder, and a timer. The recording altimeter is used to record the static pressure measured by the static-pressure source. The airspeed recorder is used to record the impact pressure, or the difference between total pressure and static pressure, measured by the pitot-static installation. The statoscope with the sensitive pressure recorder and timer is used to measure accurately the time rate of change of static pressure for the determination of vertical velocity near the start or end of the calibration run. The airspeed recorder, the recording altimeter, and the statoscope should be the only instruments connected to the static-pressure source and should be located as near as possible to it in order to minimize the lag of the pressure system. Provision must also be made so that the large volume of the statoscope is not continuously open to the static-pressure source because of lag considerations and that the sensitive pressure cell is not subjected to pressures beyond its limit in order to prevent damage. The magnitude of the pressure lag of the airspeed installation may be determined by methods described in reference 3. Where the lag is appreciable, corrections must be made to the measured static pressure. The thermometer is used to determine free-stream temperature. A properly shielded thermometer with a high temperature recovery factor (approaching 1.0) is recommended since it should be least affected by position on the airplane. The thermometer should also have a low lag; corrections should be made if the lag is appreciable. The attitude recorder is used to measure the attitude of the airplane and may consist of a horizon camera, a sun camera, or an attitude gyroscope. A horizon camera shooting either forward or laterally is probably the most desirable attitude recorder in localities where the horizon is not obscured by haze. The attitude gyroscope and the sun camera, however, may be more generally used. The attitude gyroscope measures the change in attitude angle. The attitude angle at some instant during the calibration must

therefore be determined from other measurements, perhaps from the statoscope measurements and from the estimated angle of attack. When a sun camera is used, the airplane should be equipped with an indicating device to enable the pilot the fly the airplane in a vertical plane with the lateral axis normal to the rays of the sun. One such device is a sun dial. The determination of the attitude angle from measurements with the sun camera is discussed in appendix B.

CALIBRATION PROCEDURE

The calibration procedure described herein may be used for level flight, climb, dive, push-down, and pull-out in a vertical plane. A calibration may be obtained in a single run for some conditions and does not require any additional survey of static pressure and temperature as do the methods of references 1 and 2.

For a dive starting with a push-down from level flight, the airplane is flown prior to the push-down at a condition (lift coefficient and Mach number) for which the airspeed calibration is known. Also, prior to the push-down, the statoscope is closed and the recording instruments are turned on. Measurements are made throughout the calibration run of impact pressure, static pressure, temperature, longitudinal acceleration, normal acceleration, attitude angle, and the change in static pressure from the pressure at which the statoscope was closed. The differential-pressure recorder of the statoscope, however, may go off scale when the differential pressure exceeds the range of the sensitive pressure cell. When the dive includes a pull-out to level flight, or a pull-out, climb, and then a push-down to level flight, it may be desirable, but not necessary, to obtain further measurements with the statoscope. The pull-out or push-down is made to a speed for which the airspeed calibration is known. The statoscope is then opened momentarily and closed. The instruments are turned off a few seconds after the statoscope is closed.

For a dive starting with a half roll and a pull-out (split-S) the instruments are turned on after the airplane has attained the desired attitude in the vertical plane and are kept on throughout the pull-out to level flight, or throughout the pull-out, climb, and push-down to level flight, whichever the case may be. The pull-out or push-down is made to a speed for which the calibration is known and the statoscope is opened momentarily and then closed. The instruments are turned off a few seconds after the statoscope is closed.

For a climb starting from level flight, the procedure is essentially the same as that for the dive starting from the push-down.

When a sun camera is used to obtain the attitude angle, the airplane must be flown with the lateral axis normal or very nearly normal to the rays of the sun. A simple sun dial mounted ahead of the canopy may be used by the pilot as an indicator for keeping the lateral axis normal to the rays of the sun. The local time when the instruments are turned on should be determined accurately. The clock that is used for this purpose may be checked against radio time signal (National Bureau of Standards radio station WWV).

When a horizon camera or an attitude gyroscope is used, the airplane must be flown in a vertical plane. The airplane, however, is not restricted to any particular vertical plane as in the case with the sun camera.

ACCURACY

The accuracy of the method depends principally on the accuracy of determining the attitude angle and the accuracy of the accelerometer measurements. The equations for the various errors are derived in appendix C and the errors in the computed quantities due to assumed errors in measured quantities are presented in figures 5 to 15. When a sun camera is used to determine the attitude angle, errors in attitude angle may arise from an error in time of the calibration, an error in longitude and latitude of the airplane, and an error in measurement of angle with the sun camera. By measuring the time of the start or end of the calibration run to within a few seconds, the error in attitude angle due to an error in the measurement of the time of the calibration may be almost eliminated (see fig. 5). The clock should be checked against an accurate source of time, perhaps against radio time signal, before or after the calibration. Because of the speed of the airplane, and hence the distance covered in a calibration run, the latitude and longitude of the airplane may vary appreciably during the run. If, however, the pilot can estimate his position to within 10 miles, the error in attitude angle can be kept to within $\pm 0.1^{\circ}$ (fig. 6). The error in the angle of sun rays relative to the longitudinal axis as measured with the sun camera need not be more than about 0.1° for a properly designed sun camera. The probable maximum error in attitude angle should be of the order of 0.1°.

A properly designed horizon camera can probably measure attitude angle to within 0.1°. When the horizon camera is shooting forward, the correction for dip of the horizon can be estimated to within $\pm 0.1^{\circ}$.

When the attitude angle is obtained from the flight-path angle and the angle of attack at one instant in the calibration run, together with the change in attitude angle as measured with an attitude gyroscope for

other times during the calibration run, the error in attitude angle may be of a larger magnitude than that for the method with which the sun camera is used. The error in the flight-path angle arises from an error in determining vertical velocity with the use of the statoscope. If the rate of change of static pressure with time is determined to an accuracy of ±0.01 inch of water per second, the error in vertical velocity at 40,000-feet altitude is about 3 feet per second. This error in vertical velocity for a flight Mach number of about 0.6 corresponds to an error of about 0.3° in flight-path angle. The error in estimating the angle of attack at the instant when the flight-path angle is determined depends on the applicability and the accuracy of the information on which the estimate is based. The NACA attitude gyroscopes have a drift of about 3° per minute. For a calibration run lasting, for example, 20 seconds, the error in attitude angle due to the drift of the gyroscope would be of the order of 1° at the end of the calibration run. The probable maximum error in attitude angle (assuming a 0.3° error in estimating the angle of attack) for a calibration run lasting 20 seconds would vary from about 0.4° at the start to about 1.1° at the end.

The error in the vertical component of acceleration due to an error of $\pm 1^{\circ}$ in attitude angle as shown in figure 7 increases with increase in normal acceleration and with increase in attitude angle. In dives, normal acceleration would vary from about 1.0g near level flight to 0 in a vertical dive. The error in vertical acceleration in a dive would probably be of the order of 0.01g for an error of $\pm 1^{\circ}$ in attitude angle. In a pull-out, the error in vertical acceleration would be larger but would probably occur only near the end of the calibration run.

The error in the vertical component of acceleration due to an error of ±0.01g in the normal and longitudinal components of acceleration is of the order of 0.01g (fig. 8). An NACA recording accelerometer has an accuracy of 1/4 percent of full range. A longitudinal accelerometer of $\pm\frac{1}{2}$g range and a normal accelerometer of 0 to 1 g range would have an accuracy of 0.0025g. A normal accelerometer of 0 to 4g range would have an accuracy of 0.01g. In order to get improved accuracy for normal accelerations, a combination low-range and high-range normal accelerometer can be used. For example, in a calibration run involving a dive and pull-out, a low-range normal accelerometer can be evaluated for the dive and the high-range accelerometer for the pull-out.

The error in the vertical component of acceleration due to neglecting the angle of bank varies with attitude angle and normal acceleration (fig. 9). For 1.0g normal acceleration, neglecting an angle of bank of 10° results in an error in vertical acceleration of -0.015g at zero attitude angle and of 0g in a vertical dive.

The error in the vertical component of acceleration due to neglecting the angle of yaw is shown in figure 10. For a 2° angle of yaw, the error in vertical acceleration is less than 0.01g. For transonic and supersonic speeds, an angle of yaw of 2° probably would not be unintentionally exceeded and certainly would not be maintained.

The error in static pressure due to a constant error of ± 0.01g in vertical acceleration varies as the square of the time for the calibration run (fig. 11). For a calibration run lasting 20 seconds, the error, as percent of free-stream static pressure, is 0.3, and for 40 seconds, the error is 1.1.

The error in vertical velocity due to an error in time rate of change of static pressure as determined from the statoscope is shown in figure 12. For an altitude of 40,000 feet, the error in vertical velocity is 2.8 feet per second for an error of ± 0.01 inch of water per second in the time rate of change of static pressure.

The error in static pressure due to an error of ± 0.01 inch of water per second in the time rate of change of static pressure is given in figure 13 as a function of time for an altitude of 40,000 feet. The errors in static pressure given as percent of static pressure are about 0.3 and 0.5 for calibration runs lasting 20 and 40 seconds, respectively.

The error in static pressure due to an error of ± 1 percent in measured temperature (or about $\pm 5^{\circ}$ F) varies with the range of pressures covered in the calibration (fig. 14). For a range of static pressures from 0.7 to 3 times the initial static pressure, the error is within 1 percent of free-stream static pressure.

An error in total pressure results in an error in Mach number and, hence, in static pressure as determined from equation (7). Errors in total pressure due, for instance, to angularity of flow may be avoided by the use of properly designed pitot tubes. The error in free-stream static pressure due to an error of ± 1 percent of total pressure is seen in figure 14 to be within 0.3 percent of free-stream static pressure for a range of static pressures from 0.7 to 3 times the initial static pressure.

In evaluating the free-stream static pressure with the use of equation (6), the value of the gas constant may be taken to be 53.3, the value for dry air. The resulting error due to neglecting moisture content may be shown to be negligible. For example, if the air were saturated, the use of the gas constant for dry air would introduce an error of about 0.2 percent of free-stream static pressure in a dive from an altitude of 10,000 feet to sea level and of 0.01 percent in a dive from 30,000 feet to 20,000 feet.

The error in static pressure due to error in an NACA recording altimeter is $\pm\frac{1}{4}$ percent of the full-scale reading. For an altimeter covering a range from sea level to 50,000 feet, the error in static pressure is ±1 inch of water or ±1.3 percent of static pressure at an altitude of 40,000 feet. For an altimeter covering a range of altitudes above 30,000 feet, the error is ±0.2 inch of water or ±0.3 percent of static pressure at 40,000 feet. Further improvement in accuracy of the static-pressure measurements may be obtained with the use of a statoscope equipped with a differential-pressure recorder having a range to cover the change in static pressure over the desired range of altitudes. The error in Mach number due to an error of ±1 percent of static pressure is shown in figure 15.

The elapsed time t may be measured to within 0.01 percent with the use of a tuning-fork timer. The static-pressure error corresponding to this error in time is (according to equation (45)) within 0.05 percent of free-stream static pressure for a range of altitudes $(h - h_1)$ of 50,000 feet. Since all instruments can be of the continuous recording type, no consistent error should result from the correlation of these records.

When a calibration run begins and ends near level flight at a speed for which the calibration is known, a check on the constant errors in calibration (due to errors in acceleration or attitude angle) is obtained in that the vertical velocity at the end of the calibration run should be equal to vertical velocity at the start plus the time integral of the vertical acceleration. A consistent error in the recording altimeter does not affect the determination of the static-pressure error since the consistent error would be included in the computed free-stream static pressure as well as in the measured pressure.

For the hypothetical airspeed calibration illustrated in appendix D, the probable maximum error in the determined static-pressure source was estimated on the basis of the following assumed accuracies:

$\Delta\theta$, degrees	±0.2
Δa_l (for −0.5g to 0.5g longitudinal accelerometer), g units .	±0.0025
Δa_n (for 0 to 4g normal accelerometer), g units	±0.01
Δp (for altimeter with a range including sea-level pressure), inches of water	±1.0
$\Delta\left(\frac{dp}{dt}\right)$, inches of water per second	±0.01
ΔT_m, $^{\circ}F$	±1.0

The probable maximum error in the determined static-pressure error of the static-pressure source at the end of the dive was estimated to be ±1.2 percent of free-stream static pressure. If a normal accelerometer having 0 to 1 g range and an altimeter measuring pressures only above an altitude of 25,000 feet are used, the probable maximum error is reduced to ±0.6 percent of free-stream static pressure.

CONCLUDING REMARKS

A method is described for calibrating airspeed installations on airplanes at transonic and supersonic speeds in vertical-plane maneuvers in which use is made of measurements of normal and longitudinal accelerations and attitude angle. The method involves starting or ending a calibration run near level flight at a speed for which the airspeed calibration is known and hence for which the free-stream static pressure may be determined. Integration of the vertical acceleration computed from the normal and longitudinal accelerations and the attitude angle determines the change in altitude which, when combined with the temperature measurements, gives the change in static pressure from the start or end of the calibration run and hence the variation of free-stream static pressure during the calibration run. The static-pressure error is then obtained at any instant during the calibration run by subtracting the free-stream static pressure from the indicated static pressure.

In the method described herein the required instrumentation is carried within the airplane. Should the airplane at any time enter a previously unexplored flight condition in a vertical-plane maneuver, a calibration may be readily obtained.

In measuring the attitude angle, the sun camera or the horizon camera appears to offer better accuracy than the use of an attitude gyroscope. The airplane with the sun camera, however, would be limited to a vertical plane in which the lateral axis of the airplane is normal to the rays of the sun, whereas the airplane with either the horizon camera or the attitude gyroscope could select any vertical plane. The horizon camera is restricted to use in localities where the horizon is not obscured by haze.

A study of the effect of various sources of error on the accuracy of a calibration indicated that the required quantities can be measured accurately enough to insure a satisfactory calibration.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Air Force Base, Va., February 23, 1950

APPENDIX A

CALCULATION OF SUITABLE VALUES OF n FOR USE IN EQUATION (7)

The value of n that yields zero or nearly zero error in the computed free-stream static pressure as a result of using p_m and M' in equation (7) may be found by first differentiating equation (6) as

$$\frac{\Delta p}{p} = -\frac{1}{p^n}\int \frac{p^n}{RT}\left(ne_p - e_T\right)dh \tag{11}$$

where e_p in the integral is the error in the static-pressure source and $\Delta p/p$ on the left side of the equation is the error in the computed free-stream static pressure due to use of p_m and M'. Also in the integral e_T is the error due to use of M'. For zero error in computing free-stream static pressure

$$ne_p = e_T$$

or

$$n = \frac{e_T}{e_p}$$

For $M \leqq 1.0$, the value of $\frac{e_T}{e_p}$ may be determined from equation (4) and the equation

$$p_T = p\left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \tag{12}$$

Differentiating each equation and combining results in the following expression:

$$n = \frac{e_T}{e_p} = \frac{p}{T}\frac{dT}{dp} = \frac{\gamma - 1}{\gamma}\left(\frac{1 + \frac{\gamma - 1}{2} M^2}{1 + \frac{\gamma - 1}{2} KM^2}\right) K \tag{13}$$

For $M \geqq 1.0$ the value of n may be similarly computed from equation (4) and the equation

$$p_T = \frac{\gamma + 1}{2} M^2 p \left[\frac{(\gamma + 1)^2 M^2}{4\gamma M^2 - 2(\gamma - 1)}\right]^{\frac{1}{\gamma - 1}} \tag{14}$$

or

$$n = \frac{\gamma - 1}{2\gamma} \frac{2\gamma M^2 - (\gamma - 1)}{2M^2 - 1} \frac{M^2 K}{1 + \frac{\gamma - 1}{2} KM^2} \tag{15}$$

The values of n for $K = 0.9$ and $K = 1.0$ are tabulated for various Mach numbers as follows:

M	n	
	K = 0.9	K = 1.0
0.5	0.258	0.286
1.0	.261	.286
1.5	.347	.374
2.0	.461	.490
3.0	.644	.670
4.0	.759	.779

Since, in the general case, a range of Mach number would be covered in a calibration test, a mean value of n may be taken. The possible error in the computed free-stream static pressure resulting from the use of the mean value for a range of Mach numbers from $M = 1.0$ to $M = 2.0$ was estimated to be less than 3 percent of the static-pressure error of the airspeed installation.

APPENDIX B

CALCULATION OF ATTITUDE ANGLE FROM MEASUREMENTS WITH SUN CAMERA

The attitude angle is determined by subtracting the elevation angle of the sun from the angle recorded by the sun camera

$$\theta = \alpha - \lambda \tag{16}$$

The elevation angle of the sun may be found in navigational tables by use of the date of calibration, the time at the start or end of the calibration run, and the longitude and latitude of the airplane. The elevation angle may also be found from the expression

$$\sin \lambda = \sin \epsilon \sin \tau + \cos \epsilon \cos \tau \cos(\omega - \beta) \tag{17}$$

APPENDIX C

CALCULATION OF ERRORS

Error in Attitude Angle Due to Error in Time of Calibration

An error in the time of calibration results in an error in the elevation angle of the sun and hence in the attitude angle. The error in the elevation angle of the sun is, after differentiation of equation (17),

$$\Delta\lambda = -\frac{\cos\,\epsilon\,\cos\,\tau}{\cos\,\lambda}\sin(\omega - \beta)\Delta\omega \tag{18}$$

or

$$\Delta\lambda = -\frac{\cos\,\epsilon\,\cos\,\tau}{\cos\,\lambda}\sin(\omega - \beta)\,\frac{15}{3600}\,\Delta t \tag{19}$$

where

$$\Delta\omega = \frac{15}{3600}\,\Delta t$$

and Δt is the error in time in seconds.

Since

$$\Delta\lambda = -\Delta\theta$$

$$\Delta\theta = 0.00416\,\frac{\cos\,\epsilon\,\cos\,\tau}{\cos\,\lambda}\sin(\omega - \beta)\Delta t \tag{20}$$

The error in attitude angle due to an error of 60 seconds in time of the calibration is plotted in figure 5.

Error in Attitude Angle Due to Error in Latitude and Longitude of the Airplane

The error in attitude angle due to error in the latitude of the airplane is

$$\Delta\theta = -\Delta\lambda = \left[\frac{-\sin \epsilon \cos \tau + \cos \epsilon \sin \tau \cos(\omega - \beta)}{\cos \lambda}\right] \Delta\tau \qquad (21)$$

or

$$\Delta\theta = \left[\frac{-\sin \epsilon \cos \tau + \cos \epsilon \sin \tau \cos(\omega - \beta)}{\cos \lambda}\right] \frac{57.3}{4000} s_\beta \qquad (22)$$

where

$$\Delta\tau = \frac{57.3}{4000} s_\beta$$

and s_β is in miles.

Similarly, the error in attitude angle due to error in the longitude of the airplane is

$$\Delta\theta = \frac{-\cos \epsilon \sin(\omega - \beta)}{\cos \lambda} \frac{57.3}{4000} s_\tau \qquad (23)$$

where s_τ is in miles.

The error in attitude angle due to error in the latitude and longitude of the airplane is plotted in figure 6.

Error in Vertical Component of Acceleration Due to Error in Attitude Angle

The error in vertical acceleration due to an error in attitude angle is, after differentiation of equation (10),

$$\Delta a_V = -(a_Z \sin \theta + a_X \cos \theta)\Delta\theta \tag{24}$$

where $\Delta\theta$ is in radians. The error in the vertical component of acceleration for a $\pm 1^{\circ}$ error in attitude angle is shown in figure 7.

Error in Vertical Component of Acceleration Due to Error in Longitudinal and Normal Components of Acceleration

The error in the vertical component of acceleration due to error in the longitudinal component of acceleration is, after differentiation of equation (10),

$$\Delta a_V = -\Delta a_X \sin \theta \tag{25}$$

Similarly, for an error in the normal component of acceleration

$$\Delta a_V = \Delta a_Z \cos \theta \tag{26}$$

The error in the vertical component of acceleration due to an error of $\pm 0.01g$ in the longitudinal and normal components of acceleration is presented in figure 8.

Error in Vertical Component of Acceleration Due to Angle of Bank

The expression for vertical acceleration, including the angle of bank, is

$$a_V = a_Z \cos \theta \cos \phi - a_X \sin \theta - g \tag{27}$$

where ϕ is the angle of bank.

If the angle of bank is neglected, the error is

$$\Delta a_V = a_Z \cos \theta(\cos \phi - 1) \tag{28}$$

The error in the vertical component of acceleration is plotted in figure 9 for angles of bank of 5°, 10°, and 15°.

Error in Vertical Component of Acceleration Due to Angle of Yaw

The vertical component of acceleration, including the effect of yaw, is

$$a_V = a_Z \cos \theta - a_X \sin \theta \cos \psi + a_y \sin \psi \sin \theta - g \qquad (29)$$

where ψ is the angle of yaw and a_y is the lateral acceleration due to yaw.

If the angle of yaw is neglected, the error is

$$\begin{aligned} \Delta a_V &= a_x \sin \theta(1 - \cos \psi) + a_y \sin \psi \sin \theta \\ &= \left[a_x(1 - \cos \psi) + a_y \sin \psi\right]\sin \theta \qquad (30) \end{aligned}$$

This error in the vertical component of acceleration is shown in figure 10 for 2° and 4° of yaw.

Error in Free-Stream Static Pressure Due to Error in Determining Vertical Component of Acceleration

The error in free-stream static pressure due to error in the vertical component of acceleration is found by substituting equation (8) into equation (7) and then differentiating the resulting equation, or

$$\frac{\Delta p}{p} = -\frac{1}{p^n} \int_{t_1}^{t} \left[\frac{p^n(1 + 0.2KM^2)}{RT_m} \int_{t_1}^{t} \Delta a_V \, dt\right] dt \qquad (31)$$

In the integral Δa_V, p, M, and T_m may vary with time. In order to obtain the order of magnitude of the error, however, it is sufficient to assume these quantities as constant. Then

$$\frac{\Delta p}{p} = -\frac{\Delta a_v}{2RT} t^2 \tag{32}$$

The variation of the error in static pressure with time is shown in figure 11 for an error in vertical acceleration of ± 0.01g.

Error in Vertical Velocity Due to Error in Determining the Time Rate of Change of Static Pressure

The error in vertical velocity due to error in determining the time rate of change of static pressure is obtained from equation (9) as

$$\Delta v = -\frac{RT}{p} \Delta\left(\frac{dp}{dt}\right) \tag{33}$$

The error in vertical velocity due to an error of 0.01 inch of water per second in the time rate of change of static pressure is shown in figure 12 for various altitudes.

Error in Free-Stream Static Pressure Due to Error in Determining Vertical Velocity or the Time Rate of Change of Static Pressure

The error in free-stream static pressure due to error in determining vertical velocity is, after substitution of equation (8) into equation (7) and differentiation of the resulting equation,

$$\frac{\Delta p}{p} = -\frac{1}{p^n} \int_{t_1}^{t} \frac{p^n(1 + 0.2KM^2)}{RT_m} \Delta v_1 \, dt \tag{34}$$

In evaluating the order of magnitude of error in static pressure, T_m, M, and p may be assumed as constant in the integral and therefore

$$\frac{\Delta p}{p} = -\frac{\Delta vt}{RT} \tag{35}$$

In terms of error in time rate of change of static pressure, the error in static pressure is, after substitution for Δv from equation (33),

$$\frac{\Delta p}{p} = \Delta\left(\frac{dp}{dt}\right)\frac{t}{p} \tag{36}$$

This static-pressure error is plotted in figure 13 as a function of time for an altitude of about 40,000 feet and an error of ± 0.01 inch of water in the time rate of change of static pressure.

Error in Free-Stream Static Pressure Due to Error in Measuring T_m

The error in free-stream static pressure due to error in measuring T_m is, after differentiation of equation (7),

$$\frac{\Delta p}{p} = \frac{1}{p^n}\int_{p_1}^{p} \frac{p^n(1 + 0.2KM^2)}{RT_m} \frac{\Delta T_m}{T_m}\, dh \tag{37}$$

For $\frac{\Delta T_m}{T_m}$ = Constant, equation (37) reduces to

$$\frac{\Delta p}{p} = \frac{\Delta T_m}{nT_m}\left[\left(\frac{p_1}{p}\right)^n - 1\right] \tag{38}$$

The error in free-stream static pressure due to an error of ± 1 percent in T_m is plotted in figure 14 against the ratio of initial pressure p_1 to pressure p at any time during the calibration. A value of n of $\frac{\gamma - 1}{\gamma}$ or 0.286 was assumed.

Error in Free-Stream Static Pressure Due to Error in Measuring Total Pressure

The error in total pressure introduces an error in the computation of temperature and hence in the computation of free-stream static pressure. The static-pressure error may be found by differentiating equation (6) as

$$\frac{\Delta p}{p} = \frac{1}{p^n} \int_{h_1}^{h} \frac{p^n}{RT} \frac{\Delta T}{T} \, dh \tag{39}$$

If $\frac{\Delta T}{T}$ is assumed to be a constant and is related to $\frac{\Delta p_T}{p_T}$ through the use of equations (4), (12), and (14), the static-pressure error is found to be

$$\frac{\Delta p}{p} = -\frac{\Delta p_T}{p_T} \left[\left(\frac{p_1}{p} \right)^n - 1 \right] \tag{40}$$

The error in static pressure due to ± 1-percent error in total pressure is plotted in figure 14 against the ratio of initial pressure p_1 to pressure p at any time during the calibration for $n = \frac{\gamma - 1}{\gamma}$.

Error in Free-Stream Static Pressure Due to Error in Elapsed Time

Integration of equation (8) between the limits of h and h_1 yields

$$h - h_1 = v_1 t + \int_0^t \int_0^t a_v \, dt \, dt \tag{41}$$

If the measured elapsed time t_m is substituted into equation (41) the equation becomes

$$h_m - h_1 = v_1 t_m + \int_0^{t_m} \int_0^{t_m} a_v \, dt_m \, dt_m \tag{42}$$

If the error in elapsed time is defined as

$$e_t = \frac{t_m - t}{t}$$

the error in altitude determined from equations (41) and (42) becomes

$$\Delta h = e_t \left[2(h - h_1) - v_1 t \right] \tag{43}$$

The corresponding error in free-stream static pressure is

$$\frac{\Delta p}{p} = \frac{e_t}{RT} \left[v_1 t - 2(h - h_1) \right] \tag{44}$$

The error in free-stream static pressure is a maximum if v_1 is zero or if v_1 has a direction opposite to the resultant change in altitude. For $v_1 = 0$,

$$\frac{\Delta p}{p} = \frac{2e_t(h_1 - h)}{RT} \tag{45}$$

Error in Mach Number Due to Error in Free-Stream Static Pressure

The error in Mach number due to an error in static pressure is, after differentiation of equation (12) for $M \leqq 1.0$,

$$\Delta M = -\frac{1 + 0.2M^2}{1.4M} \frac{\Delta p}{p} \tag{46}$$

and, after differentiation of equation (14) for $M \geqq 1.0$,

$$\Delta M = -\frac{M}{7} \frac{7M^2 - 1}{2M^2 - 1} \frac{\Delta p}{p} \tag{47}$$

The error in Mach number due to a ±1-percent error in free-stream static pressure is shown in figure 15.

APPENDIX D

ILLUSTRATION OF METHOD WITH THE USE OF HYPOTHETICAL DATA

In the hypothetical airspeed calibration, the airplane is assumed to be equipped with a sun camera for the determination of the attitude angle. The following data, pertinent to the determination of the elevation or altitude angle of the sun, are assumed:

Date	April 25, 1949
Time at start of calibration	13^h00^m E.S.T. or 18^h00^m G.C.T.
Longitude of Langley Laboratory, β	$76^\circ 21'$
Latitude of Langley Laboratory, τ	$37^\circ 5'$

From navigational tables, the Greenwich hour angle ω is found to be $90^\circ 31'$ and the declination ϵ is $13^\circ 15'$. With these values of longitude, latitude, Greenwich hour angle, and declination, the elevation angle of the sun λ is found from navigational tables or computed from the expression (see appendix B)

$$\sin \lambda = \sin \epsilon \sin \tau + \cos \epsilon \cos \tau \cos(\omega - \beta)$$

to be 65.4°. The attitude angle of the airplane is then obtained as

$$\theta = \alpha - \lambda$$

$$= \alpha - 65.4^\circ$$

where α is the angle between the rays of the sun and the longitudinal axis as measured with the sun camera. Assumed values of a_z, a_x, and α for various times during the calibration are given in figure 1 and table 1. The computations for determining a_v are also indicated in table 1.

The assumed variation of p_T, p_m, and T_m with time is shown in figure 2 and table 2. Computation of temperature T' for a temperature recovery factor of the thermometer of 0.99 is also shown in table 2.

The initial vertical velocity v_1 is computed from the data assumed for the statoscope pressure recorder as shown in figure 3. The

static-pressure error e_p at the start of the dive is assumed to be known and to have the value 0.02. With the use of this value of static-pressure error, the statoscope data may be corrected as follows:

$$\Delta p = \Delta p_m(1 - e_p)$$

This correction, however, is negligible for the present example. From a plot of Δp against t, the slope dp/dt at t = 0 is found to be 0.04 inch of water per second. The initial vertical velocity v_1 is then found from the expression

$$v_1 = -\frac{RT'_1}{p_1}\left(\frac{dp}{dt}\right)_1$$

where

$$T'_1 = 394.4 \text{ (table 2)}$$

$$p_1 = p_{m_1}(1 - e_p) \text{ (table 2)}$$

$$= 76.8\ (1 - 0.02) = 75.3 \text{ inches of water}$$

and

$$R = 53.3$$

Therefore

$$v_1 = -11.2 \text{ feet per second}$$

The computation of the static-pressure error $\frac{p_m - p}{p}$ of the airspeed installation is indicated in table 3. The static-pressure error is also plotted in figure 4 against indicated free-stream Mach number.

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3. Huston, Wilber B.: Accuracy of Airspeed Measurements and Flight Calibration Procedures. NACA Rep. 919, 1948.

TABLE 1.– COMPUTATION OF VERTICAL ACCELERATION FROM ATTITUDE-ANGLE AND ACCELERATION DATA FOR HYPOTHETICAL CALIBRATION

[All accelerations in g units and angles in deg]

t	a_z	a_x	α	$\alpha - \lambda$	$\cos(\alpha - \lambda)$	$\sin(\alpha - \lambda)$	$a_z \cos(\alpha - \lambda)$	$a_x \sin(\alpha - \lambda)$	$a_z \cos(\alpha - \lambda)$ minus $a_x \sin(\alpha - \lambda)$	a_v (a)
0	0.996	0.089	61.3	–4.1	0.9974	–0.0715	0.9934	–0.0064	0.9998	0
1	.996	.089	61.3	–4.1	.9974	–.0715	.9934	–.0064	.9998	0
2	.996	.089	61.3	–4.1	.9974	–.0715	.9934	–.0064	.9998	0
3	.500	.011	64.7	–.7	.9999	–.0122	.5000	–.0001	.5001	–.500
4	0	.152	69.4	4.0	.9976	.0698	0	.0106	–.0106	–1.011
5	0	.160	72.7	7.3	.9919	.1271	0	.0203	–.0203	–1.020
6	0	.159	75.8	10.4	.9836	.1805	0	.0287	–.0287	–1.029
7	0	.157	78.8	13.4	.9728	.2317	0	.0364	–.0364	–1.036
8	0	.156	81.9	16.5	.9588	.2840	0	.0443	–.0443	–1.044
9	0	.155	84.9	19.5	.9426	.3338	0	.0517	–.0517	–1.052
10	0	.155	87.9	22.5	.9236	.3827	0	.0593	–.0593	–1.059
11	0	.151	90.8	25.4	.9033	.4289	0	.0648	–.0648	–1.065
12	0	.148	93.6	28.2	.8813	.4726	0	.0699	–.0699	–1.070
13	0	.145	96.2	30.8	.8600	.5120	0	.0742	–.0742	–1.074
14	0	.141	98.7	33.3	.8358	.5490	0	.0774	–.0774	–1.077
15	.190	.140	99.7	34.3	.8261	.5635	.1570	.0789	.0781	–.922
16	.632	.145	99.0	33.6	.8329	.5534	.5264	.0802	.4462	–.554
17	.861	.147	98.2	32.8	.8406	.5417	.7238	.0796	.6442	–.356
18	.860	.140	98.2	32.8	.8406	.5417	.7229	.0758	.6471	–.353
19	.860	.137	98.5	33.1	.8377	.5461	.7204	.0748	.6456	–.354
20	.861	.126	98.6	33.2	.8368	.5476	.7205	.0690	.6515	–.349
21	.862	.120	98.8	33.4	.8349	.5505	.7197	.0661	.6536	–.346
22	.862	.112	98.9	33.5	.8339	.5519	.7188	.0618	.6570	–.343
23	.863	.102	99.0	33.6	.8329	.5534	.7188	.0564	.6624	–.338
24	.864	.091	99.1	33.7	.8319	.5548	.7188	.0505	.6683	–.332
25	.865	.030	99.2	33.8	.8310	.5563	.7188	.0167	.7021	–.298
26	.867	–.035	99.2	33.8	.8310	.5563	.7205	–.0195	.7400	–.260
27	.868	–.100	99.3	33.9	.8300	.5578	.7204	–.0558	.7762	–.224
28	.870	–.163	99.4	34.0	.8290	.5592	.7212	–.0911	.8123	–.188
29	.871	–.232	99.4	34.0	.8290	.5592	.7221	–.1297	.8518	–.148
30	.872	–.293	99.4	34.0	.8290	.5592	.7229	–.1638	.8867	–.113
31	.872	–.330	99.5	34.1	.8281	.5606	.7221	–.1850	.9071	–.093
32	.872	–.355	99.5	34.1	.8281	.5606	.7221	–.1990	.9211	–.079
33	.872	–.370	99.5	34.1	.8281	.5606	.7221	–.2074	.9295	–.071
34	.872	–.386	99.6	34.2	.8271	.5621	.7212	–.2170	.9382	–.062

[a] a_v is preceding column minus 1.000.

TABLE 2.– COMPUTATION OF INDICATED MACH NUMBER AND APPROXIMATE FREE-STREAM TEMPERATURE FOR HYPOTHETICAL CALIBRATION FROM PRESSURE AND TEMPERATURE DATA

[All pressures in inches of water and temperature in °F abs.]

t	p_T (a)	p_m (a)	$\frac{p_T}{p_m}$	M'	$\frac{T_m}{T'}$ (b)	T_m (a)	T'
0	96.2	76.8	1.253	0.577	1.066	420.4	394.4
1	96.2	76.8	1.253	.577	1.066	420.4	394.4
2	96.2	76.9	1.251	.575	1.066	420.4	394.4
3	96.2	76.9	1.251	.575	1.066	420.4	394.4
4	96.3	77.1	1.249	.573	1.065	420.4	394.7
5	97.0	77.3	1.255	.579	1.066	420.7	394.7
6	97.9	77.6	1.262	.586	1.068	421.3	394.5
7	99.0	78.1	1.268	.592	1.069	421.9	394.7
8	100.2	78.7	1.273	.598	1.071	422.6	394.6
9	101.8	79.4	1.282	.607	1.073	423.5	394.7
10	103.8	80.2	1.294	.618	1.076	424.5	394.5
11	106.1	81.2	1.307	.630	1.079	425.5	394.3
12	108.8	82.5	1.319	.642	1.082	426.8	394.5
13	111.9	83.8	1.335	.656	1.085	428.3	394.7
14	115.5	85.4	1.352	.671	1.089	430.0	394.9
15	119.5	87.0	1.374	.689	1.094	431.8	394.7
16	124.3	88.9	1.398	.709	1.100	434.1	394.6
17	129.5	90.7	1.428	.732	1.106	436.8	394.9
18	135.1	92.9	1.454	.751	1.112	439.7	395.4
19	141.0	95.0	1.484	.773	1.118	442.9	396.2
20	147.0	97.4	1.509	.790	1.124	446.3	397.1
21	153.3	99.9	1.535	.807	1.129	450.0	398.6
22	160.0	102.4	1.563	.825	1.135	454.0	400.0
23	167.2	105.0	1.592	.843	1.141	458.2	401.6
24	174.6	107.8	1.620	.860	1.146	462.9	403.9
25	182.5	110.6	1.650	.877	1.152	467.6	405.9
26	190.8	113.6	1.680	.894	1.158	472.5	408.0
27	199.0	116.7	1.705	.907	1.163	476.9	410.1
28	207.0	120.0	1.725	.918	1.167	481.1	412.3
29	214.7	123.3	1.741	.926	1.170	485.0	414.5
30	222.1	126.6	1.754	.933	1.172	488.5	416.8
31	229.5	130.0	1.765	.939	1.175	491.8	418.6
32	236.8	133.5	1.774	.943	1.176	495.0	420.9
33	243.7	137.0	1.779	.946	1.177	497.8	422.9
34	250.5	140.6	1.782	.947	1.178	500.5	424.9

[a]From recorded data. See also figure 2.

[b] $\frac{T_m}{T'} = 1 + 0.2KM'^2 = 1 + 0.198M'^2$ where $K = 0.99$.

TABLE 3.– CALCULATION OF FREE-STREAM STATIC PRESSURE AND STATIC-PRESSURE ERROR

[All pressures in inches of water, acceleration in g units and temperature in °F abs.]

t	$-a_v$	$-\int a_v\,dt$	$\frac{-v_1}{g} - \int_0^t a_v\,dt$ (a)	$-\frac{1}{gT'}\frac{dh}{dt}$	$\frac{p_m}{p_1}$ (b)	$\left(\frac{p_m}{p_1}\right)^{0.286}$	$-\frac{1}{RT'}\left(\frac{p_m}{p_1}\right)^{0.286}\frac{dh}{dt}$	$-0.286\int_0^t \left(\frac{p_m}{p_1}\right)^{0.286}\frac{dh}{dt}\frac{dt}{RT'}$	$\frac{p}{p_1}$	$\frac{p_m - p}{p_m}$
0	0	0	0.348	0.000882	1.020	1.006	0.00054	0	1.000	0.020
1	0	0	.348	.000882	1.020	1.006	.00054	.0002	1.001	.019
2	0	0	.348	.000882	1.021	1.006	.00054	.0003	1.001	.020
3	.500	.250	.598	.001516	1.021	1.006	.00092	.0005	1.002	.019
4	1.011	1.006	1.354	.003430	1.024	1.007	.00209	.0009	1.003	.020
5	1.020	2.021	2.369	.006002	1.026	1.007	.00365	.0018	1.006	.020
6	1.029	3.046	3.394	.008603	1.030	1.008	.00524	.0030	1.011	.019
7	1.036	4.078	4.426	.011214	1.037	1.010	.00684	.0048	1.017	.019
8	1.044	5.118	5.466	.013852	1.045	1.013	.00848	.0070	1.025	.019
9	1.052	6.166	6.514	.016504	1.054	1.015	.01012	.0096	1.034	.019
10	1.059	7.222	7.570	.019189	1.065	1.018	.01180	.0128	1.046	.018
11	1.065	8.284	8.632	.021892	1.078	1.022	.01352	.0164	1.059	.018
12	1.070	9.351	9.699	.024586	1.095	1.026	.01524	.0205	1.074	.020
13	1.074	10.423	10.771	.027289	1.112	1.031	.01700	.0251	1.091	.020
14	1.077	11.498	11.846	.029997	1.134	1.038	.01881	.0302	1.110	.021
15	.922	12.498	12.846	.032546	1.155	1.042	.02049	.0358	1.131	.021
16	.554	13.236	13.584	.034425	1.180	1.048	.02180	.0419	1.154	.023
17	.356	13.691	14.039	.035551	1.204	1.054	.02264	.0482	1.179	.022
18	.353	14.046	14.394	.036404	1.233	1.062	.02336	.0548	1.205	.024
19	.354	14.399	14.747	.037221	1.261	1.068	.02402	.0616	1.233	.022
20	.349	14.750	15.098	.038021	1.294	1.077	.02474	.0686	1.261	.025
21	.346	15.098	15.446	.038751	1.326	1.084	.02538	.0757	1.291	.027
22	.343	15.442	15.790	.039475	1.360	1.092	.02604	.0831	1.322	.028
23	.338	15.783	16.131	.040167	1.395	1.100	.02669	.0906	1.354	.029
24	.332	16.118	16.466	.040768	1.431	1.108	.02729	.0983	1.388	.031
25	.298	16.433	16.781	.041343	1.470	1.116	.02787	.1062	1.424	.031
26	.260	16.712	17.060	.041814	1.510	1.125	.02842	.1143	1.460	.032
27	.224	16.954	17.302	.042190	1.550	1.133	.02888	.1225	1.498	.034
28	.188	17.160	17.508	.042464	1.594	1.142	.02930	.1308	1.538	.035
29	.148	17.328	17.676	.042644	1.638	1.151	.02965	.1392	1.578	.036
30	.113	17.458	17.806	.042721	1.682	1.160	.02994	.1477	1.620	.036
31	.093	17.562	17.910	.042785	1.726	1.169	.03022	.1564	1.663	.036
32	.079	17.648	17.996	.042756	1.774	1.178	.03043	.1650	1.707	.037
33	.071	17.722	18.070	.042729	1.820	1.187	.03064	.1738	1.752	.037
34	.062	17.789	18.137	.042685	1.868	1.196	.03084	.1825	1.798	.037

[a] $-\frac{v_1}{g} - \int_0^t a_v\,dt = \frac{1}{g}\frac{dh}{dt}$.

[b] $p_1 = p_{m_1}(1 - e_p) = 76.8\ (1 - 0.02) = 75.3$.

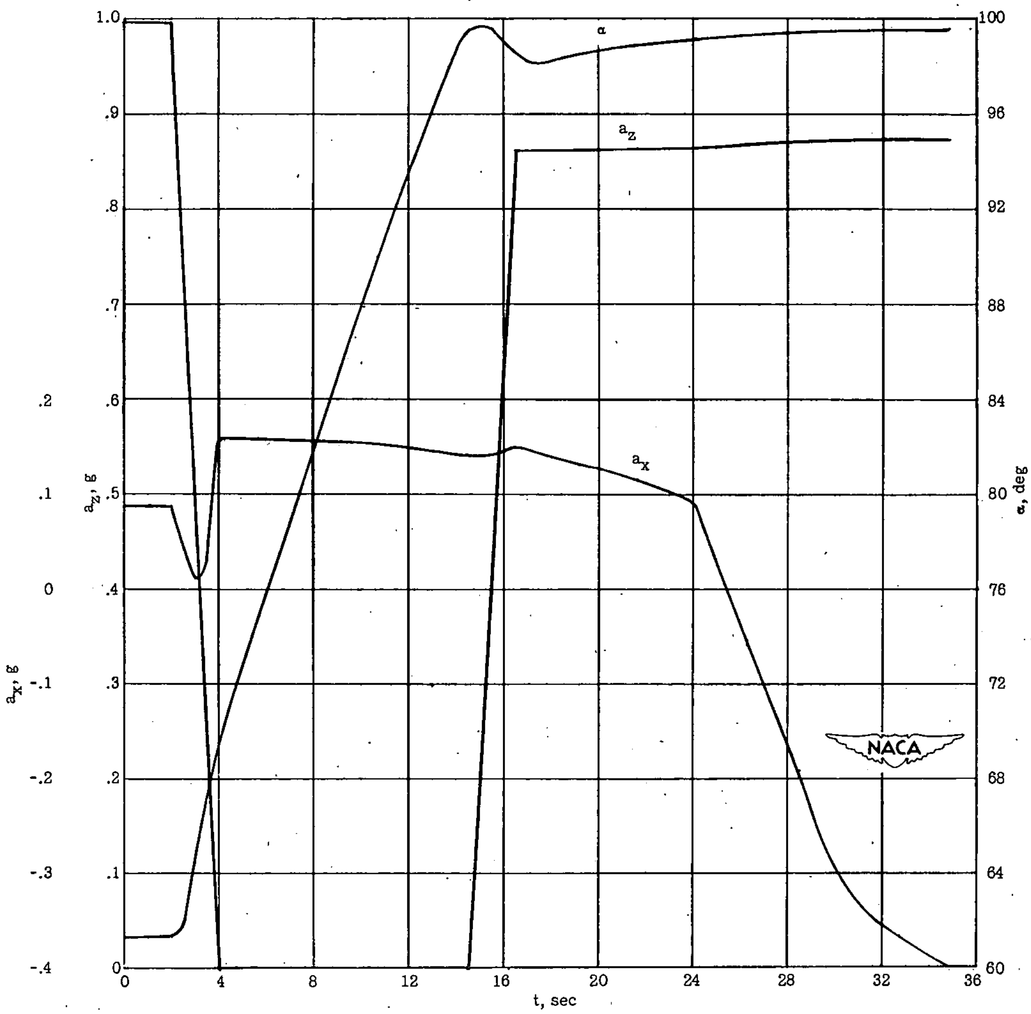


Figure 1.- Time history of longitudinal acceleration, normal acceleration, and angle measured by sun camera for hypothetical calibration. See table 1.

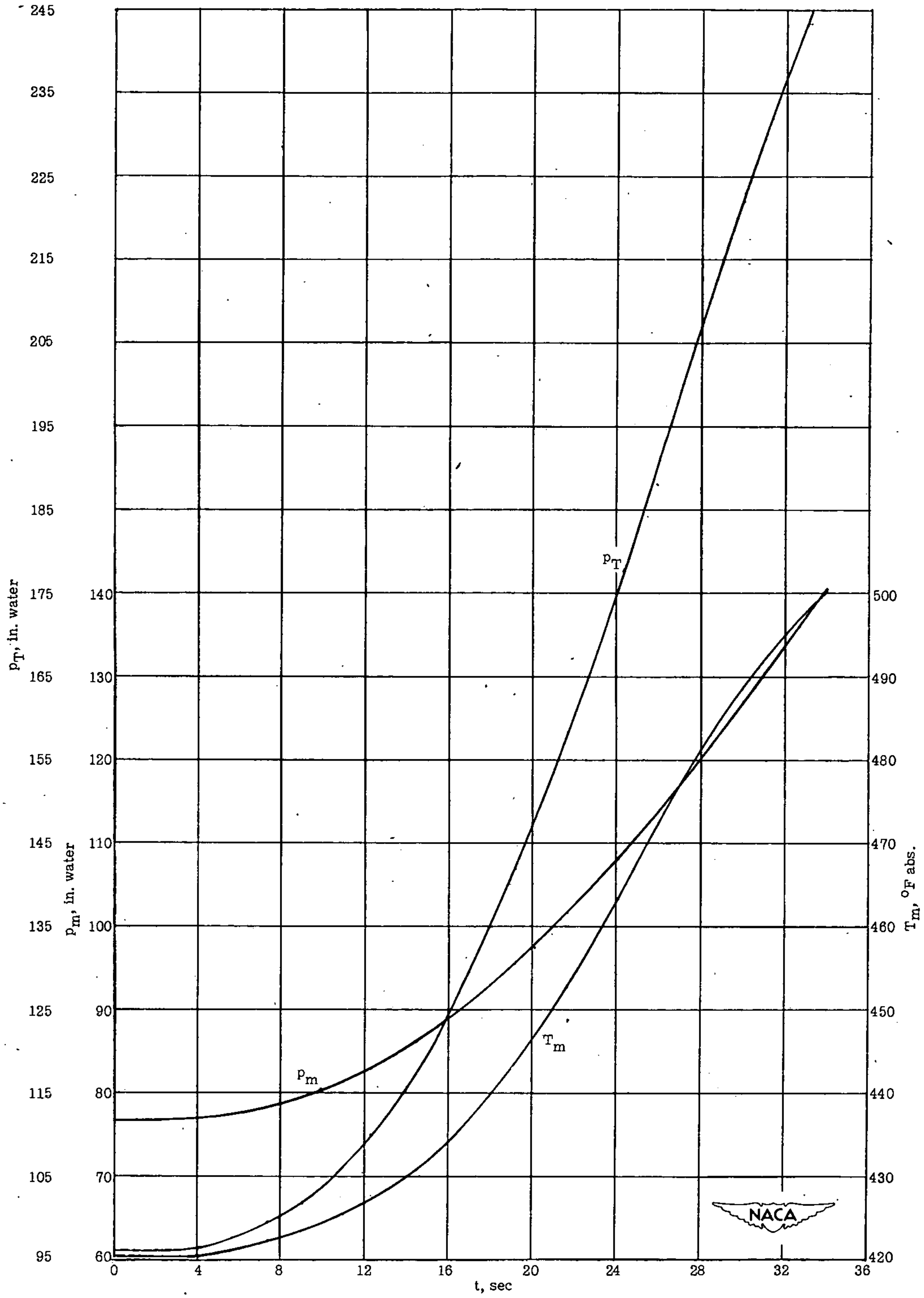


Figure 2.- Time history of total pressure, indicated free-stream static pressure, and indicated temperature for hypothetical calibration. See table 2.

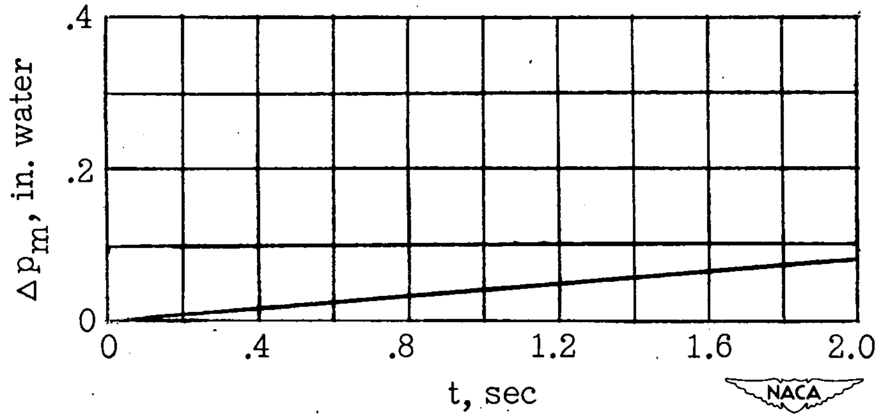


Figure 3.- Assumed variation with time of change in static pressure at start of hypothetical calibration. $\Delta p_m = p_m - p_{m_1}$.

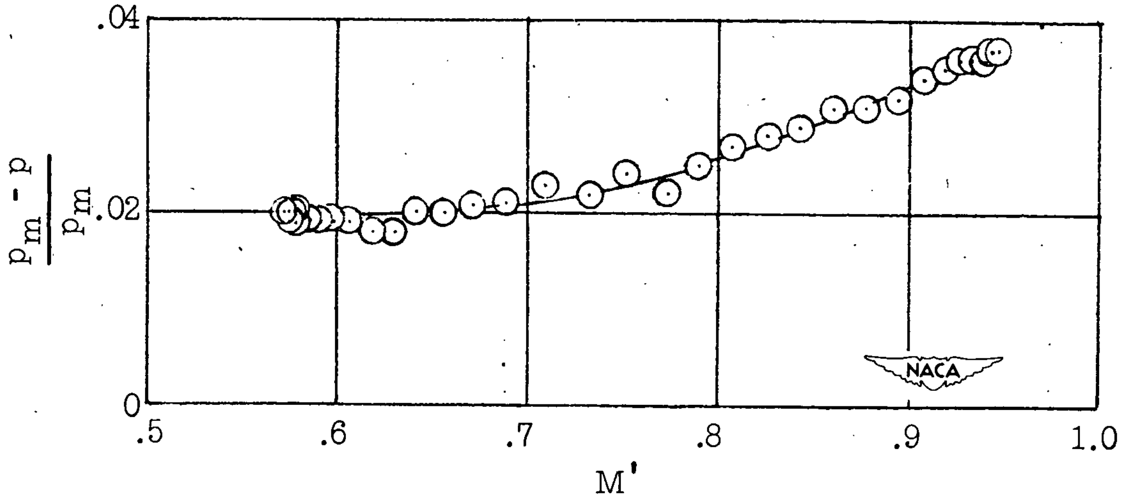


Figure 4.- Static-pressure error for hypothetical calibration. See table 3.

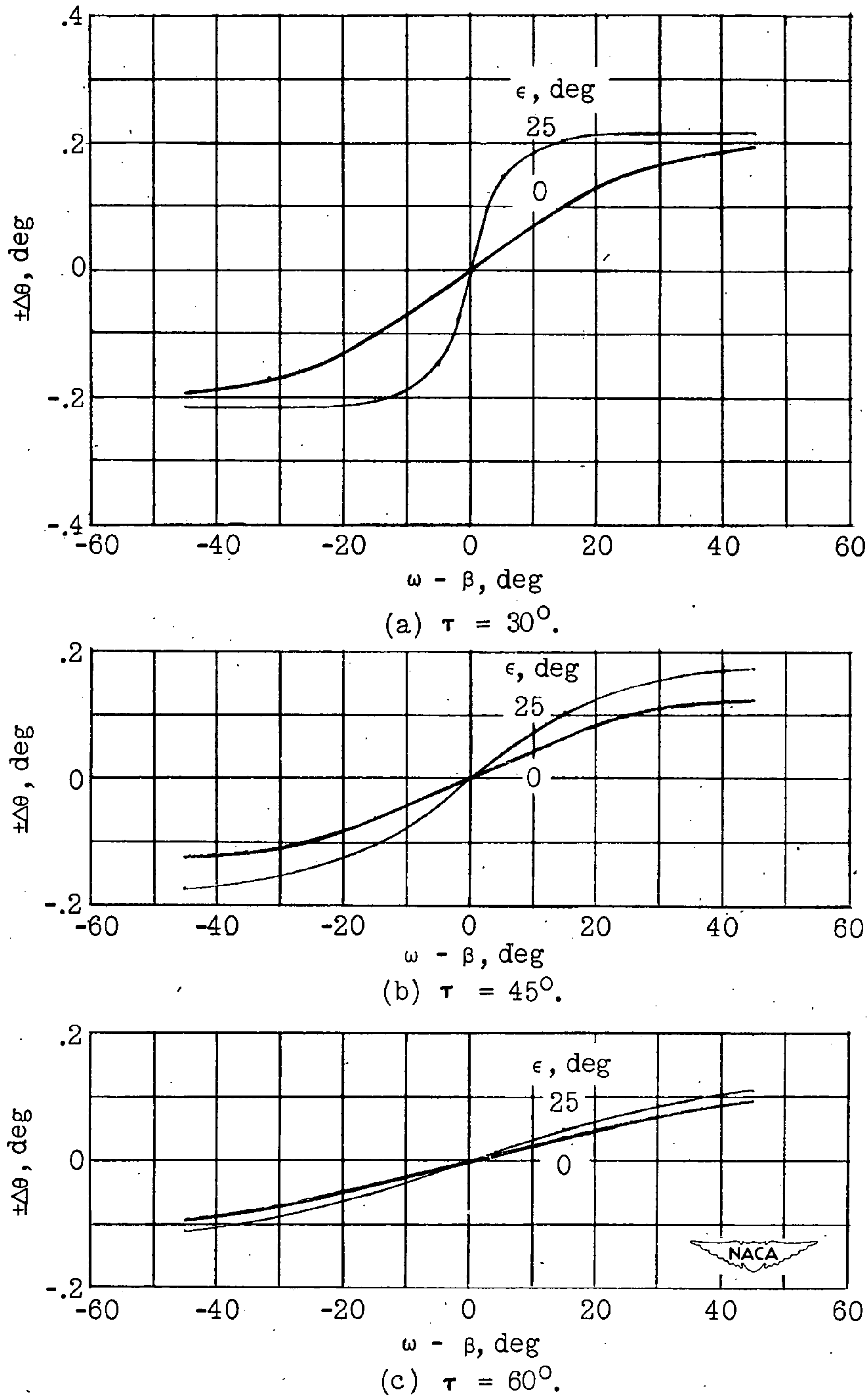
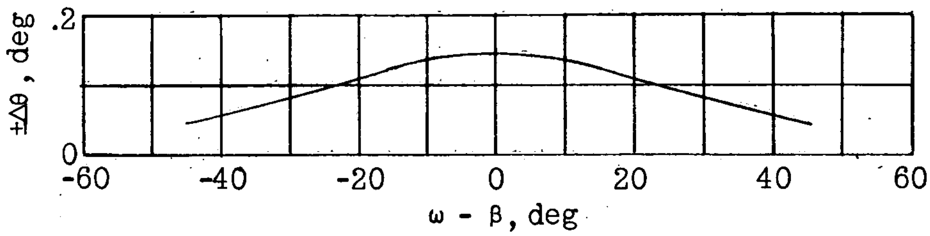
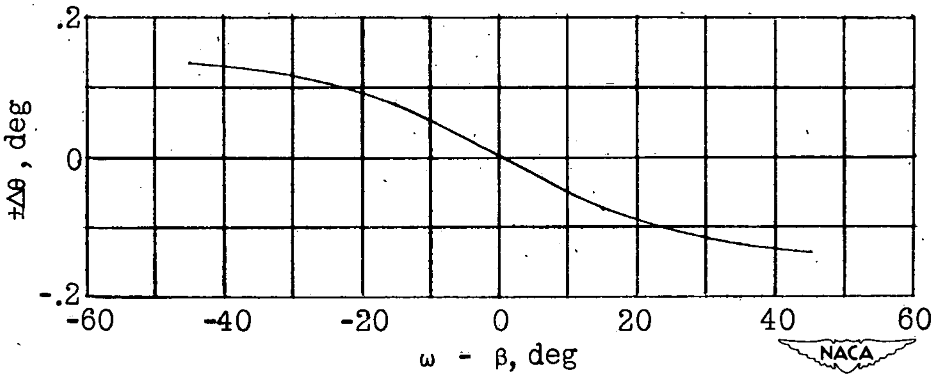


Figure 5.- Error in attitude angle due to error of ±60 seconds in determining local time of calibration when use is made of a sun camera.



(a) Due to error of ±10 miles in latitude.



(b) Due to error of ±10 miles in longitude.

Figure 6.- Error in attitude angle due to error in estimating position of airplane. $\epsilon = 15^{\circ}$; $\tau = 40^{\circ}$.

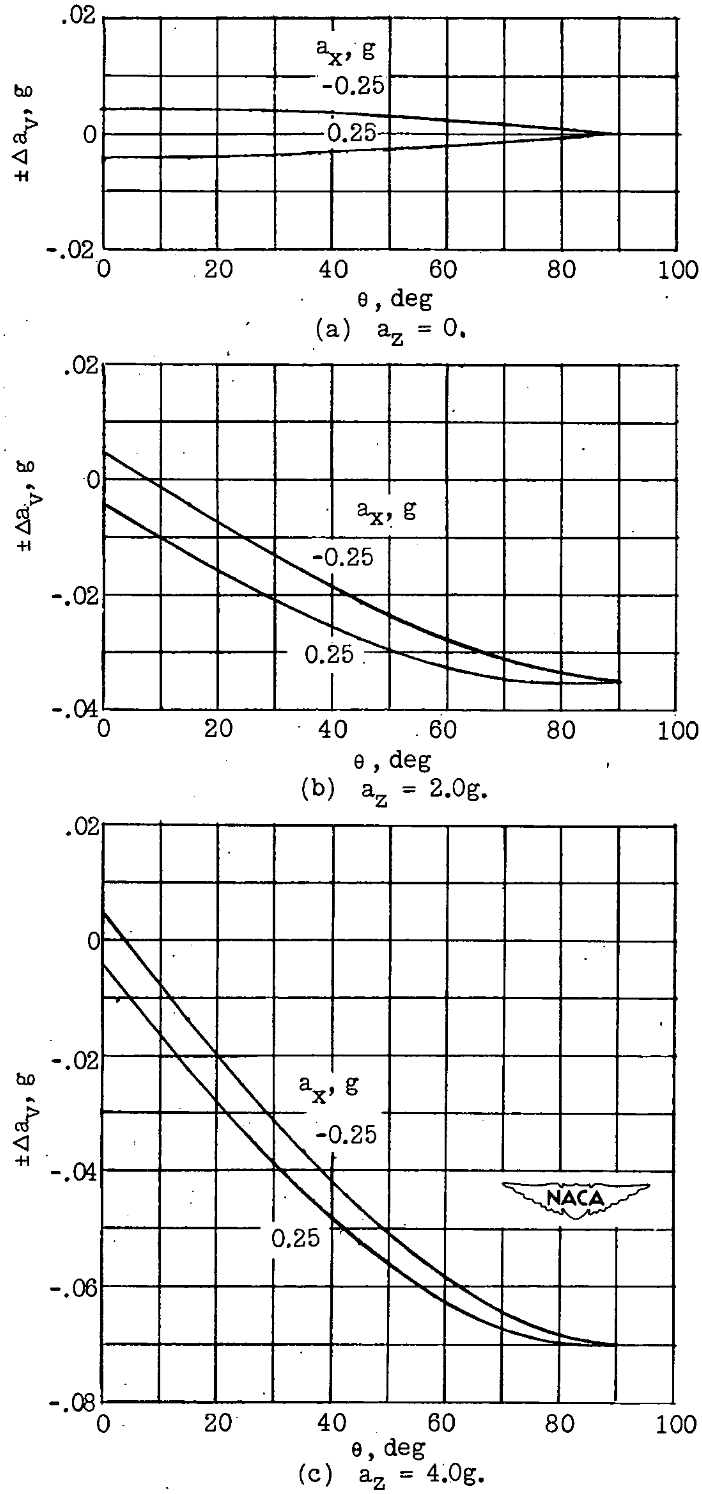
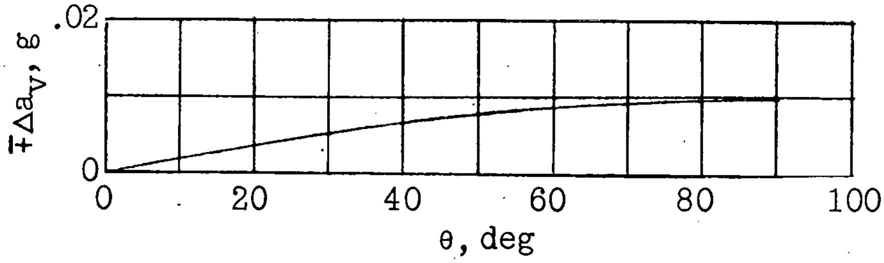
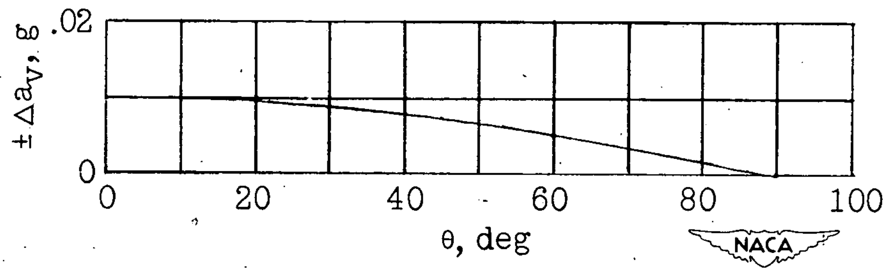


Figure 7.- Error in vertical component of acceleration due to ±1° error in attitude angle.

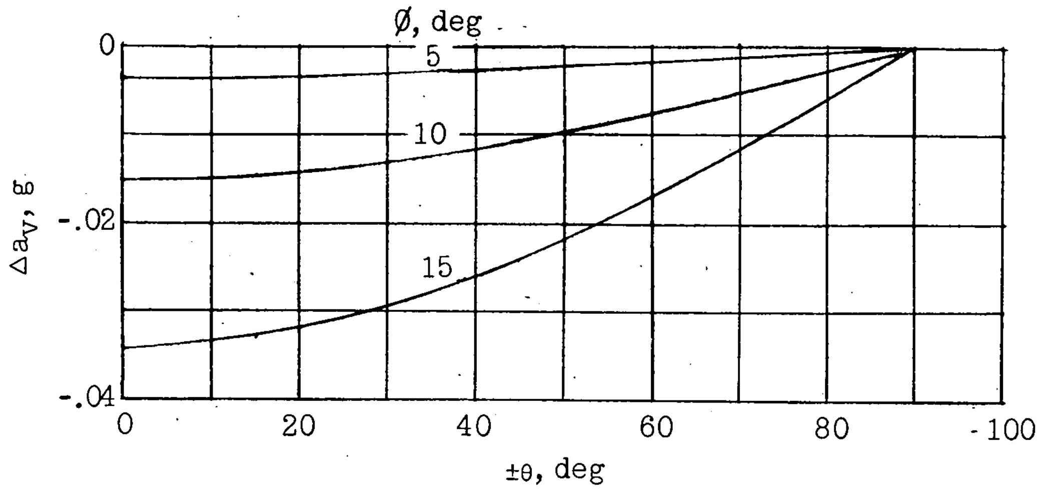


(a) Due to error in longitudinal acceleration.

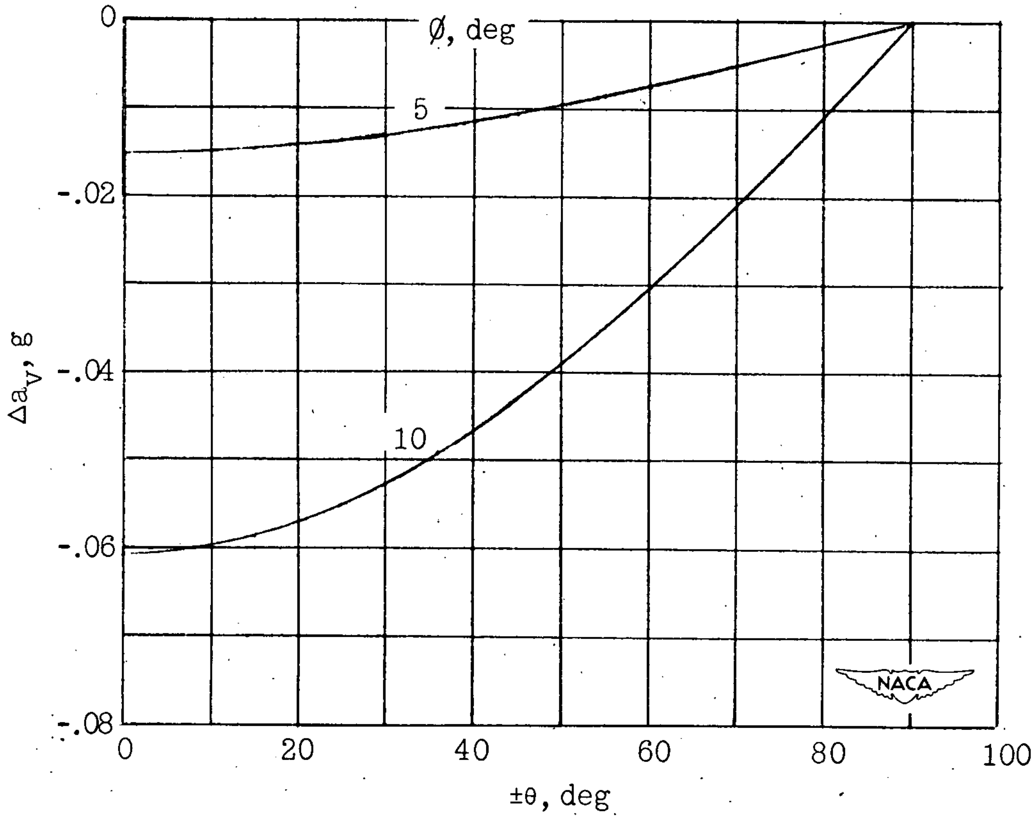


(b) Due to error in normal acceleration.

Figure 8.- Error in vertical component of acceleration due to an error of ±0.01g in longitudinal and normal accelerations.



(a) $a_z = 1.0g$.



(b) $a_z = 4.0g$.

Figure 9.- Error in vertical component of acceleration due to neglecting angle of bank.

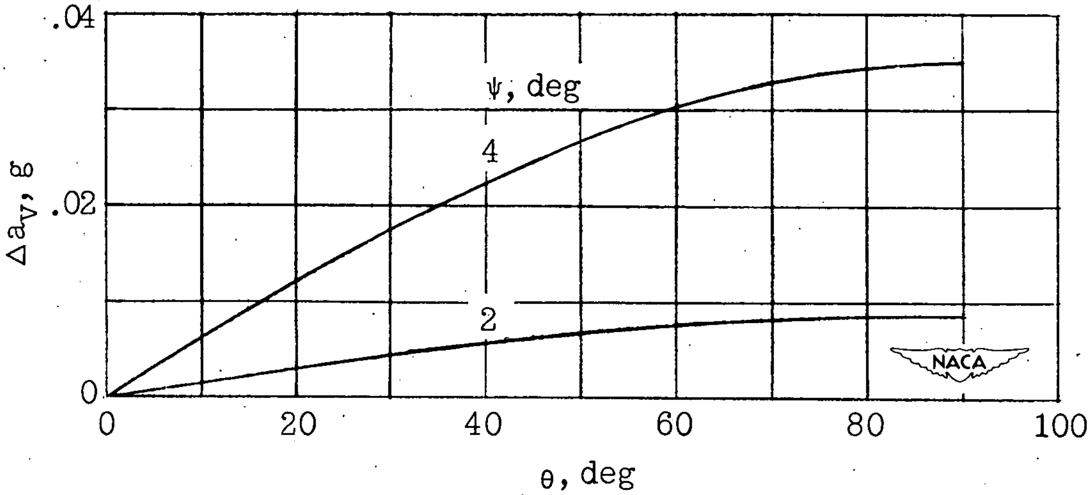


Figure 10.- Error in vertical component of acceleration due to neglect of angle of yaw. $a_y = 0.25g$ and $0.50g$ for $\psi = 2^\circ$ and 4°, respectively. $a_x = 0$.

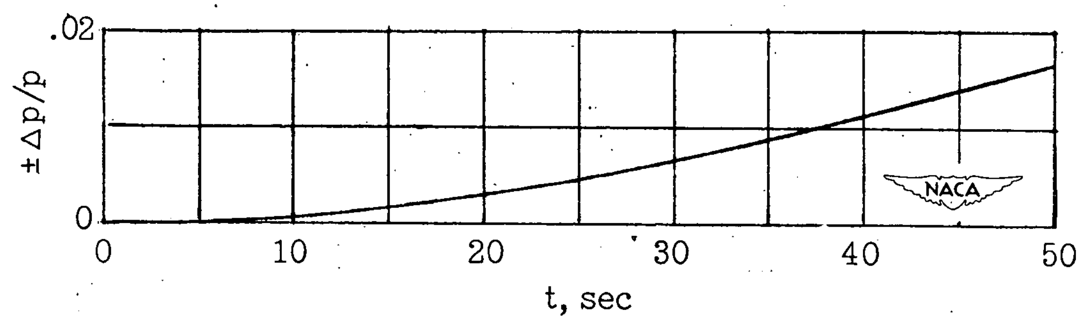


Figure 11.- Error in free-stream static pressure due to consistent error of ±0.01g in vertical component of acceleration.

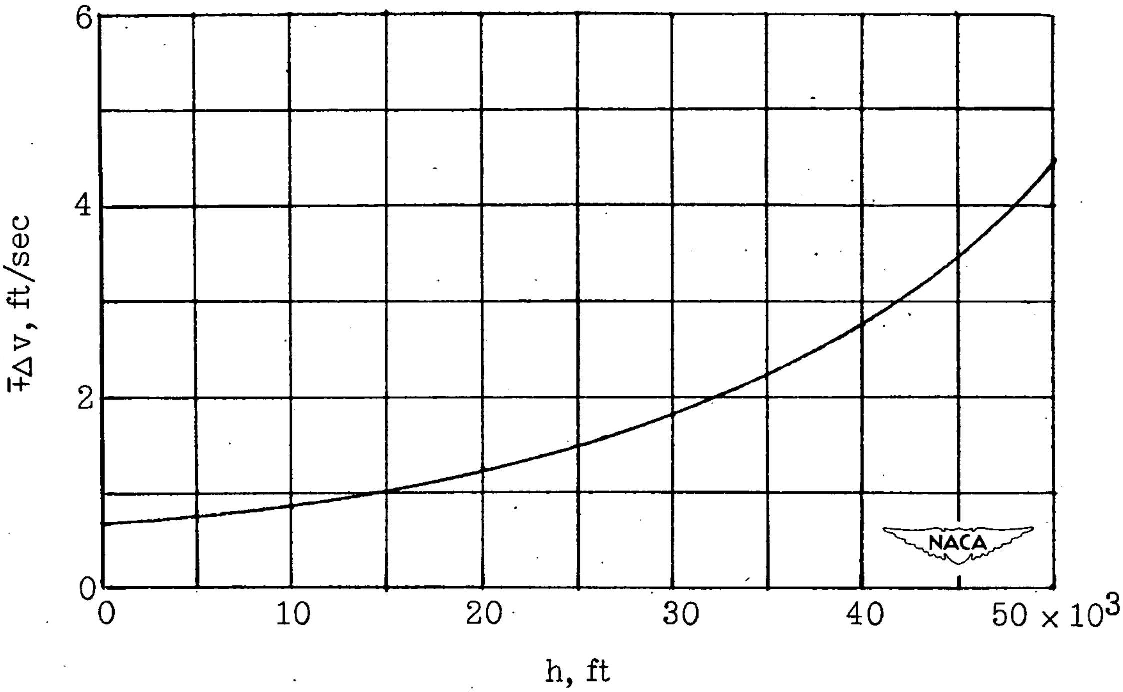


Figure 12.- Error in vertical velocity due to error of ±0.01 inch of water per second in time rate of change of static pressure.

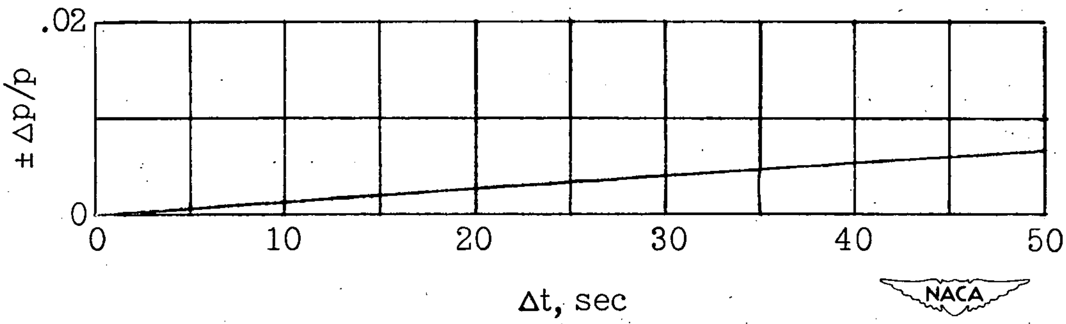


Figure 13.- Error in static pressure due to error of ±0.01 inch of water per second in time rate of change of static pressure. $h \approx 40,000$ feet.

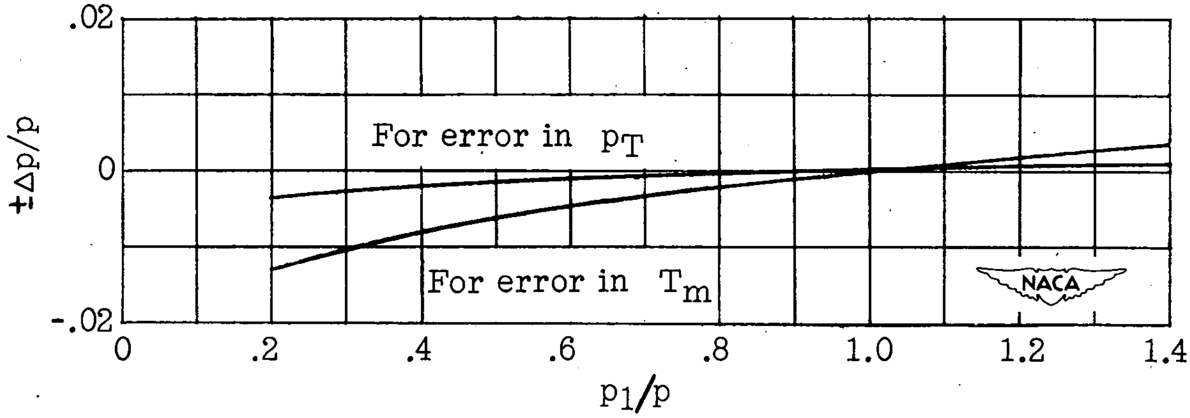


Figure 14.- Error in free-stream static pressure due to ±1-percent error in measured temperature and ±1-percent error in total pressure.

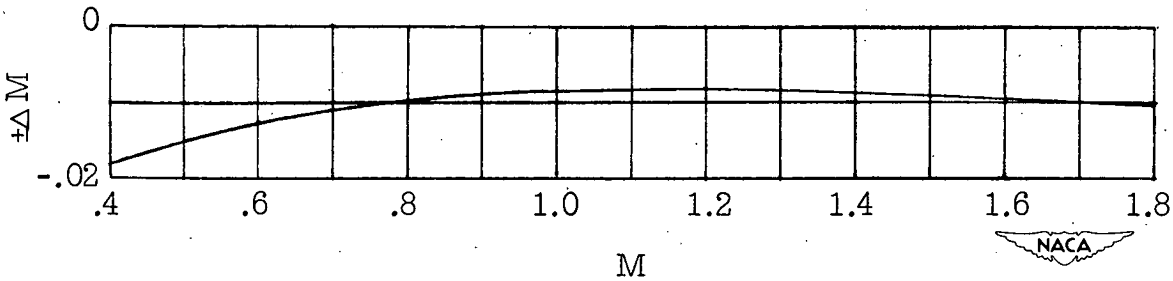


Figure 15.- Error in Mach number corresponding to ±1-percent error in static pressure.